we treat instead a "boxed" oscillator in much the same sense as a "boxed" particle in elementary electron theory. This follows both from the point of view of our *a priori* physical knowledge as to the *finite* limits of a material atomic oscillator, and the simple fact that an oscillator in either classical or quantal theory can be termed simple harmonic *only* for *small* displacements. As to the latter, it seems rather meaningless to ascribe specific heat anomalies to anharmonicity in view of the infinite domain accessible to the Planck oscillator, which must be considered as simple harmonic *only by definition.*

The eigenvalues of a bounded oscillator in the first approximation are found to be

$$E_n = n^2h^2/32ma^2 - m\omega^2a^2/n^2\pi^2 + m\omega^2a^2/6, \quad (1)^*$$

where the boundary conditions are the vanishing of ψ at $\pm a$, and the unperturbed problem is obviously the corresponding boxed particle. This part of the problem is rather trivial, and one may readily verify by the variation theorem that the correction term is as above; $O(a^2)$. These eigenvalues lead to rather involved thermodynamic functions, which however can be approximately evaluated either in terms of Error or Bessel functions. Both methods were used, as was the classical sum of states which is here given since it is useful for asymptotic considerations ($T\rightarrow\infty$).†

$$Q = \left(\frac{kT}{\hbar\omega}\right) erf(z), \quad E = \frac{kT}{2}\left\{2 - z\frac{d}{dz}\ln erf(z)\right\},$$
$$z = a(m\omega^2/2kT)^{\frac{1}{2}}, \quad E_\infty = kT/2, \quad C_{v\infty} = k/2. \quad (2)$$

We note first of all that this result is not unexpected since the "equipartition theorem" applies only to the momentum in this case, and as might be anticipated on physical grounds (cf. Eq. (1)), the particle properties dominate at high energies (temperature); hence the above asymptotic values. However, the oscillator properties are not completely washed out, and the specific heat curve for $3N$ bounded linear oscillators shows a monotonic decrease to zero in much the manner of the Debye curve. Allowing the lattice parameter to be an arbitrary function of T (varying box, which should correspond closely to calculating C_p) does not appear to make any significant difference in the conclusions obtained. The correct quantal average energy E and specific heat C_v are found to be remarkably sensitive to small variations in the lattice parameter (our a above). This might lead one to hope that further development along these lines could remove the high temperature anomaly, and hence give a theory of specific heats which introduces the lattice constants in a fundamental way.

However, it seems possible that the real issue involved is that the usual oscillator picture is entirely too literal, and that one must instead treat the elastic oscillations as an ideal "phonon" gas which is again formally described by an aggregate of ideal oscillators as in boson theory. Such a phonon gas is no less real or fictitious than the boson gas, from which it differs only in having a finite frequency spectrum (defined by the modes of the lattice), and longitudinal vibrations. All other properties such as creation, annihilation, non-conservation of numbers, Compton effect, etc., are common to both types of particles. This approach to the problem appears to eliminate most of the conceptual difficulties and leads back to the issue of proper frequency distributions, but it prohibits the interpretation of anomalies in terms of anharmonicity.

* A Morse potential adds a great many terms to the energy, but the essential dependence on the quantum number n, is the same.
† The quantal expressions are not given because they are so lengthy and involved, but they lead to the same conclusions for asymptotic T. Note also in Eqs. (2) that $E\rightarrow kT$, $C_V\rightarrow k$, as $a\rightarrow\infty$.

Centrifugal Fields*

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RECENT experiments[1,2] on the production of high centrifugal fields have been extended using rotors with somewhat smaller diameters. The various sized rotors have similar spherical shapes and are made of hardened steel. They were suspended magnetically in a vacuum and spun by a rotating magnetic field using an apparatus previously described.[2] Table I shows the new results obtained with the smallest rotor so far tried together with some results previously reported for comparison. The rotational speed measurements were made just before rotor explosion.

TABLE I.

Diameter of rotor mm	Rotor speed r.p.s.	Peripheral speed cm/sec.	Centrifugal acceleration
1.59	211,000	1.05×10^5	1.43×10^8 g
0.795	386,000	9.65×10^4	2.40×10^8 g
0.521	633,000	1.04×10^5	4.28×10^8 g

It will be observed that each rotor exploded at a peripheral speed of approximately 10^5 cm/sec. Since the rotors were all hard steel and had similar shapes this result is in accord with theory which shows that in a spinning homogeneous elastic rotor the maximum stress produced is proportional to $(2\pi N)^2r^2 = v^2$ where N is the number of revolutions per sec., r is the rotor radius, and v is the peripheral speed. In order to measure the rotor speed,[2] one-half of the steel rotor was darkened by dipping it into dilute H_2SO_4 which had been in contact with metallic antimony. Microscopic examination of some of the broken pieces shows that small patches of this coating apparently were ripped off the periphery of the 0.521-mm rotor where this coating was too thick. From the table it will be observed that this rotor attained a speed of 633,000 r.p.s. which gave a centrifugal field of 428 million times gravity.

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